Nominal Debt as a Burden to Monetary Policy*

Giorgia Giovannetti **Ramon Marimon**
Pedro Teles†

September, 1999

Abstract

We study a dynamic equilibrium model where the same optimal monetary policy is implemented with and without full commitment if government debt is indexed. In contrast, with nominal debt, the full commitment policy is time inconsistent, since the government is tempted to inflate away its nominal liabilities. We characterize the optimal sequential policy. It has the feature that government debt is progressively depleted, and so, eventually, the time inconsistency problem vanishes. We compare this equilibrium to a myopic solution and to the Ramsey solution.

1 Introduction:

The purpose of this paper is to address the question of how debt, and in particular **nominal debt,** can affect the optimal sequential choice of monetary policy. Our aim is to identify the mechanisms at work within an equilibrium monetary model, similar to the one studied in Marimon, Nicolini and

*Preliminary and incomplete. Please do not quote. We owe a lot to Juan Pablo Nicolini in particular to Nicolini (1997). We also thank Isabel Correia for comments. Errors are ours.

†**G. Giovannetti**: Università di Firenze and European University Institute; **R. Marimon**: European University Institute, Universitat Pompeu Fabra, CEPR and NBER, and **P. Teles**: Banco de Portugal and Universidade Católica Portuguesa.

Teles (1994, 1997), Chari and Kehoe (1999) and Giovannetti, Marimon and Teles (1998), using a simplified framework and emphasizing the importance of strategic interaction.

The model is simple, but has a rich structure because it allows for costs of an unexpected inflation through the timing of the cash in advance constraint as in Svensson (1985). Due to this timing, even if the government is not able to commit to its monetary policy, there is an interior solution for inflation, as shown in Nicolini (1997).

We analyze different regimes, considering cases in which there is full commitment of the government, or central bank, and cases where there is no commitment. We consider different equilibrium concepts. We analyze as a benchmark the equilibrium with indexed debt. In this case, because of our particular structure, time inconsistency is not an issue and the optimal policy is to keep the nominal interest rate constant. With nominal debt, on the other hand, the full commitment solution is time inconsistent, in the sense that if the government was to reoptimize at a later date, still being able to commit, would choose a different policy. The government is tempted to inflate away its liabilities (even if not in one period). Hence, the existence of nominal debt brings in distortions to monetary policy. We also compute the markov perfect equilibrium, named recursive. The interesting feature of this markov perfect equilibrium is that the interest rate is not constant, but decreasing over time because the debt is being depleted. Hence, while it starts above the interest rate prevailing in the indexed debt case, in the limit it is lower (converging to the interest rate of the case of zero debt). The decreasing path for the interest rates indicates that the existence of nominal debt is indeed a burden for monetary policy. Hence, the main result is that the policy under the recursive equilibrium is non stationary and it converges to the solution with indexed debt, as debt is depleted to zero. Asymptotically there is no time inconsistency.

We do not allow for alternative taxes, i.e. the only tax that the government is allowed to use is the inflation tax. Notice that if the solution is stationary, then there is a unique interest rate that balances the budget. Since both the solution with indexed debt and the myopic solution are stationary, they do indeed share the same policy for the interest rate. However, the multipliers of the government budget constraints are different in these two optimal taxation problems, allowing us to understand the burden of the debt and to infer the solution with alternative taxes. The same reasoning applies to the solution in the recursive equilibrium: from the optimal values of the multipliers we can infer the results with alternative taxes.

The issue of how the fiscal policy and the level of debt may affect monetary policy is not new. However, recently many different proposals have been put forward which were designed as if fiscal discipline was a prerequisite for monetary (price) stability. The budget balance proposal in the US and the Maastricht Treaty and later the Stability and Growth Pact in Europe are examples of such proposals. At the policy level the issue was of whether (and sometimes what kind of) constraints should be imposed on fiscal policies of a monetary union member state. For instance, the Maastricht Treaty establishes two parallel mechanisms that seem mutually inconsistent, although they may reinforce each other. The first can be seen in the design of monetary institutions that should, on their own merits, guarantee *a commitment to a policy of price stability.* For example, rules regarding ECB independence, constraints on seignorage or ECB rent-sharing, etc., are part of such mechanism. The second, as if showing from the beginning a lack of confidence in the first mechanism, corresponds to the Stability Pact as a constraint on fiscal policies within the EMU (as well as to the Maastricht fiscal criteria as constraints for entrance in the EMU before it started in January 1999).

At the theoretical level, the issue has been mainly dealt with independently of the commitment/no commitment issue. Examples are the unpleasant monetarist arithmetic of Sargent and Wallace (1981), and the fiscal theory of the price level of Sims (1994) and Woodford (1996). In these approaches policies are exogenous. This is not the case here, or in recent related literature such as Chari and Kehoe (1999), which consider the role of nominal debt in designing monetary policy in a monetary union, thus accounting for strategic behavior within the time consistent literature. However, they do not fully addressed the issue.

The general structure of the model is presented in Section 2 where the competitive solution is spelled out. Different classes of equilibria are then considered. We start from the standard case of indexed debt, which coincides with considering real debt (Section 3). In Section 4, we compute equilibria without commitment. In Section 5 we compute a myopic equilibrium and the Ramsey equilibrium. The Ramsey policy introduces an issue of timing, because the marginal cost of taxation at time $t = 0$ is very different from that prevailing from time $t = 1$, since the government has the possibility of making a surprise, or so it is perceived. In Section 6 we compare the paths of realized interest rates (and multipliers) under the different policies we have considered, emphasising the differences between the solutions for the markov perfect equilibrium, the Ramsey, and a myopic equilibrium, where the government follows a myopic policy in the sense that the effects of the policy on future policy variables are not accounted for. Section 7 contains the conclusions. Appendix 1 details the calculations of the algorithm.

2 The Model and the competitive equilibria

The economy is composed of a continuum of identical infinitely-lived households and the government. The households have preferences given by:

$$W = \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{1}$$

where c_t is the consumption of a private good in period t and n_t the amount of labor. The utility function u shares the usual assumptions of concavity and differentiability. The households jointly consume a public good g. In each period, labor is used to produce either the private good or the public good, according to the resources constraints:

$$c_t + g \leq n_t,\ t \geq 0 \tag{2}$$

The representative household chooses sequences of consumption of the private good and leisure that satisfy the following budget constraints:

$$M_{t+1} + B_{t+1} \leq M_t + B_t(1 + i_t) - p_t c_t + p_t n_t,\ t \geq 0 \tag{3}$$

together with a non-Ponzi games condition. M_{t+1} and B_{t+1} are the nominal money balances and government bonds holdings from period t to period $t+1$ i.e. the nominal value of assets held, i_t is the nominal rate of return on government's one-period debt in period t so that $B_t(1 + i_t)$ is the value of nominal debt held from the preceding period.

Consumption goods in period t have to be purchased using the money balances carried from the previous period. That is, the consumer is also subject to a cash-in-advance constraint of the form[1]

$$p_t c_t \leq M_t,\ t \geq 0 \tag{4}$$

[1]Notice that such -Svensson (1985)'s type- cash-in-advance constraint differs from the more standard -Lucas's type- constraint where it is possible to choose M_0 and B_0. See Nicolini (1997) for an ehaustive discussion of the different implications of these constraints.

Hence, $M_t - p_t c_t$ in (3) is the unspent cash.

Divide through the budget constraint by p_t to get the budget constraint in real terms:

$$m_{t+1}(1+\pi_{t+1}) + \frac{B_{t+1}}{p_t} \leq m_t - c_t + \frac{B_t}{p_t}(1+i_t) + n_t \tag{5}$$

where m_t are real balances in period t and $\pi_t = \frac{p_t}{p_{t-1}} - 1$ is the inflation rate.

The initial stock of Money, M_{-1} and the initial stock of debt, B_{-1} are given.

The **government** issues money and debt to finance an exogenous constant level of public goods. Hence it decides on when and how to collect the inflation tax. Abstracting (for the moment) from alternative sources of tax revenues, the budget constraint of the government is given by:

$$M_{t+1} + B_{t+1} \leq M_t + B_t(1+i_t) + p_t g,\ t \geq 0 \tag{6}$$

together with a no-Ponzi games condition. The second term on the right hand side of (6) is the payment on debt incurred in the previous period, while the last term is the payment for government consumption.

An equilibrium for this economy must satisfy the standard condition equating the marginal rate of substitution between consumption and leisure to the interest rate:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, t \geq 0 \tag{7}$$

Given that preferences are linear in leisure, the real interest rate is the rate of time preference

$$\beta^{-1} = (1+i_{t+1})\frac{p_t}{p_{t+1}}, t \geq 0 \tag{8}$$

Furthermore, whenever $i_{t+1} > 0$, the cash-in-advance constraint is binding, i.e.,

$$c_{t+1} = \frac{M_{t+1}}{p_{t+1}} \tag{9}$$

From now on, for simplicity, we concentrate on the case $u(c) = \log(c)$.

It is convenient to describe the stationary competitive equilibrium in the case in which there is no inherited debt, i.e. $B_t = 0$.[2]

[2] As we shall see, in this case, i.e. when there are no outstanding liabilities, the solution of the government problem is time consistent.

For given sequences of money (and nominal debt), a competitive equilibrium is a set of allocations for consumers and a set of prices such that (i) consumers solve their maximization problem, (ii) the government budget constraint is satisfied, (iii) the resources constraints are satisfied and (iv) the money (and debt) market clear. In this equilibrium the consumers maximize (1) subject to (3), (4). Money earns a gross nominal return of 1. Hence, in any equilibrium, the gross interest rate is bounded below by 1, otherwise consumers could make a profit by buying money and selling bonds.

Using the expressions derived for the consumers maximization problem above (i.e.(7), (8), (9), together with the resources constraints (2) and the government constraint (6), we get:

$$m(1+\pi) = n, \tag{10}$$

$$(1+i) = (1+\pi)\beta^{-1}, \tag{11}$$

$$c = \frac{1}{\alpha(1+i)}, \tag{12}$$

$$c = m, \tag{13}$$

$$c + g = n.$$

The above equations reduce to:

$$n = \frac{\beta}{\alpha} \tag{14}$$

$$m = c = \frac{\beta}{\alpha} - g \tag{15}$$

$$1 + i = \frac{1}{\beta - \alpha g} \tag{16}$$

$$1 + \pi = \frac{\beta}{\beta - \alpha g} \tag{17}$$

allowing us to identify univocally the equilibrium levels of labor, money, consumption, the interest rate and the inflation rate. In this case, by substituting

($c = m = \frac{\beta}{\alpha} - g$) into the utility function we can write the one period steady state utility as:

$$\overline{u} = [\log(\frac{\beta}{\alpha} - g) - \beta]$$

3 Equilibria with indexed debt

We address the case of indexed (real) debt[3] as a benchmark, mainly to contrast it with the case we are interested in, i.e. when the debt is in nominal terms (not indexed).

As we shall see in what follows, the assumption of the log utility is important for our results. Another important assumption is linearity in leisure. Because of the assumption of linearity, the type of *fiscal* time inconsistency identified by Lucas and Stokey (1983) is not present here. Because of the assumption of log utility, in the absence of nominal debt, the optimal taxation problem is time consistent. The intuition is that the elasticity of the consumption of the good at time 0 with respect to the price level and the intertemporal elasticity of substitution coincide.[4] In this case all the goods are to be taxed equally. Nominal debt introduces gains from the initial lump sum, that raise again the issue of time inconsistency. This is discussed in Nicolini (1997).

Let $q_t \equiv 1/p_t$ be the price of money and $b_t \equiv B_t q_{t-1}$be the real debt.

Definition 1 *A Ramsey equilibrium is a policy such that:*

The government chooses prices q_t as a function of the two state variables, nominal money and real debt by solving:

$$V^I(M_t, b_t) = \max_{(q_t, b_{t+1}, M_{t+1})} \left\{ \ln(q_t M_t) - \alpha(q_t M_t + g) + \beta V^I(M_{t+1}, b_{t+1}) \right\}$$

subject to the implementability conditions:[5]

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - \beta^{-1} b_t - g = 0 \tag{18}$$

[3] This problem is indexed by I.

[4] At time zero the cash in advance is binding, and so the elasticity with respect to the price level is one.

[5] The literature refers to these conditions as implementability constraints because they are constraints on the set of allocations that can be implemented as a competitive equilibrium with distorting taxes. Constraint (18) is the consumer budget constraint with prices substituted from the first order conditions.

$$M_{t+1} = \frac{\beta}{\alpha}\frac{1}{q_t} \tag{19}$$

Since the Ramsey problem with indexed debt is time consistent, in the sense that the continuation of the Ramsey policy is Ramsey, we can solve the Ramsey problem recursively. Hence, here the government chooses prices that maximize the preferences of consumers, subject to the competitive constraint and the feasibility constraint, i.e. we use a reduced form to account for all the constraints.

The marginal condition for q_t is:

$$\frac{1}{q_t} - \alpha M_t + \beta V^I_{M_{t+1}}(-\frac{\beta}{\alpha}\frac{1}{q_t^2}) + \beta V^I_{b_{t+1}} M_t = 0 \tag{20}$$

and for b_{t+1}:

$$\beta V^I_{b_{t+1}} + \psi^I_t = 0 \tag{21}$$

where V_j denotes $\frac{\partial V}{\partial j}$, $j = M, b$ and ψ^I_t is the multiplier of the implementability condition (18).

Using the envelope theorem, we get

$$V^I_{b_t} = -\psi^I_t \beta^{-1} \tag{22}$$

$$V^I_{M_t} = \frac{1}{M_t} - \alpha q_t - \psi^I_t q_t \tag{23}$$

Using (21) and (22), gives:

$$\psi^I_t = \psi^I \tag{24}$$

i.e. the multipliers are constant when debt is indexed.

Dividing through (20) by M_t, rearranging the terms using (19), dividing through by α, and using:

$$\frac{1}{\alpha q_t M_t} = 1 + i^I_T$$

we get the expression:

$$1 + i^I_t = 1 + \beta(1 + i^I_t)\left[1 - \left(1 + \frac{\psi^I_{t+1}}{\alpha}\right)\frac{1}{1 + i^I_{t+1}}\right] + \frac{\psi^I_t}{\alpha} \tag{25}$$

It is convenient to define: $z_t^I \equiv \left(1+\frac{\psi_t^I}{\alpha}\right)\frac{1}{1+i_t^I}$, where $\psi_t^I = \psi^I$, to obtain a difference equation

$$z_t^I = (1-\beta) + \beta z_{t+1}^I$$

The solution to this difference equation is

$$z_t^I = 1.$$

Notice that, otherwise, if $z_t^I > 1$, it would become arbitrarily large, meaning that the interest rate would become negative, or else, if $z_t^I < 1$, it would become negative, also meaning that the interest rate was negative.

We can now write:

$$i_t^I = \frac{\psi^I}{\alpha} \tag{26}$$

i.e., the nominal interest rate is equal to the multiplier of the implementability constraint, accounted for at the marginal value of labor. By using the present value budget constraint, which is stationary, we can easily determine also the value of the nominal interest rate and therefore the multiplier.

$$1+i^I = \frac{1}{\beta - \alpha g - \alpha b\frac{1-\beta}{\beta}}$$

where b is the stationary value of the real debt.

We have solved a very simple problem: the government has in every period only one tax (i.e. the inflation tax).[6] The solution of the maximization problem is a constant interest rate which is optimal, because the government wants to smooth distortions. What happens here is that the government internalizes the fact that it cannot default on the debt, by surprising the agents with inflation. As we shall see, whether the government can internalize or not the fact that it has to pay for the debt is the main difference with the other equilibrium concepts, and this shows in the equilibrium values of the multipliers of the implementability constraints. In the following section we analyze the equilibrium with nominal debt and no commitment.

[6] Note that a model with other taxes may have more interesting features but the structure is essentially the same.

4 Recursive Monetary Equilibria with Nominal Debt

In this section we characterize the markov perfect equilibria with nominal debt. We name this equilibrium recursive (RE). The relevant state variables are money and nominal debt at time t.

Definition 2 *The recursive monetary equilibrium is the solution of the following problem*

$$V(M_t, b_t, Q(M_t, b_t)) =$$

$$\max_{(q_t, b_{t+1}, M_{t+1})} \{\log(q_t M_t) - \alpha(q_t M_t + g) + \beta V(M_{t+1}, b_{t+1}, Q(M_{t+1}, b_{t+1}))\}$$

subject to the implementability constraints:

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - b_t \beta^{-1} \frac{q_t}{Q(M_t, b_t)} - g = 0 \tag{27}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q_t} \tag{28}$$

Let ψ_t^{RE} be the lagrange multiplier on the first (budget) implementability constraint. Let $W(M_t, b_t) \equiv V(M_t, b_t, Q(M_t, b_t))$, and so

$$W_{b_t} = V_{b_t} + V_Q(t) \frac{\partial Q}{\partial b_t}$$

$$W_{M_t} = V_{M_t} + V_Q(t) \frac{\partial Q}{\partial M_t}$$

From the first order conditions for q_t, we get:

$$\frac{1}{q_t} - \alpha M_t + \beta W_{M_{t+1}} \left(-\frac{\beta}{\alpha} \frac{1}{q_t^2} \right) + \beta W_{b_{t+1}} \left[M_t + b_t \beta^{-1} \frac{1}{Q(M_t, b_t)} \right] = 0 \tag{29}$$

and for b_{t+1}:

$$\beta W_{b_{t+1}} + \psi_t^{RE} = 0 \tag{30}$$

Using the envelope theorem, we get the values of the marginal derivatives with respect to Q_t, b_t and M_t respectively:

$$V_Q(t) = \psi_t^{RE} b_t \beta^{-1} \frac{q_t}{Q(M_t, b_t)^2} \tag{31}$$

$$V_{b_t} = -\psi_t^{RE} \beta^{-1} \frac{q_t}{Q(M_t, b_t)} \tag{32}$$

$$V_{M_t} = \frac{1}{M_t} - \alpha q_t - \psi_t^{RE} q_t \tag{33}$$

Using (30) and (31)-(32), as well as the fact that in a REE $q_t = Q(M_t, b_t)$, we get:

$$-\psi_{t+1}^{RE} + \psi_{t+1}^{RE} \frac{b_{t+1}}{q_{t+1}} \frac{\partial Q}{\partial b_{t+1}} + \psi_t^{RE} = 0$$

that is to say, if we denote the elasticity of Q with respect to the debt b by ϵ_b,we obtain the following difference equation for the multipliers and the elasticity:

$$\psi_t^{RE} = \left(1 - \epsilon_{b_{t+1}}\right) \psi_{t+1}^{RE} \tag{34}$$

Notice that if $\epsilon_{b_{t+1}} < 0$ and $\epsilon_{b_{t+1}} \to 0$, as the debt approaches zero, then the multipliers decline and converge to a constant. If $b_t = 0$, $\epsilon_{b_{t+1}} = \frac{b_{t+1}}{Q(M_{t+1}, b_{t+1})} = 0$. Furthermore, since

$$W_{b_t} = V_{b_t} + V_Q(t) \frac{\partial Q}{\partial b_t} = V_{b_t} - V_{b_t} \frac{b_t}{Q(M_t, b_t)} \frac{\partial Q}{\partial b_t}$$

if the elasticity converges to zero, then $W_{b_t} \to V_{b_t}$. This is clear since the problem with nominal debt is equivalent to the problem with real debt, when debt is zero.

Rearranging (29), using (30)and (31)-(33) as well as $q_t = Q(M_t, b_t)$, we get:

$$\begin{aligned} \frac{1}{q_t} &= \alpha M_t + \frac{1}{q_t} \left[\beta \left(\frac{1}{q_t M_{t+1}} - \alpha \frac{q_{t+1}}{q_t} - \psi_{t+1}^{RE} \frac{q_{t+1}}{q_t} \right) + \psi_{t+1}^{RE} \frac{b_{t+1}}{q_t M_{t+1}} \epsilon_{M_{t+1}} \right] \frac{\beta}{\alpha} + \\ &\quad + \psi_t^{RE} \left[M_t + b_t \beta^{-1} \frac{1}{q_t} \right] \end{aligned} \tag{35}$$

Dividing through by M_t and using (28)

$$\begin{aligned}\frac{1}{q_t M_t} &= \alpha + \frac{1}{q_t M_t}\left[\beta\left(\frac{\alpha}{\beta} - \alpha\frac{q_{t+1}}{q_t} - \psi_{t+1}^{RE}\frac{q_{t+1}}{q_t}\right) + \psi_{t+1}^{RE} b_{t+1}\frac{\alpha}{\beta}\epsilon_{M_{t+1}}\right]\frac{\beta}{\alpha} + \\ &\quad + \psi_t^{RE}\left(\frac{M_t + b_t\frac{\beta^{-1}}{q_t}}{M_t}\right) \qquad (36)\end{aligned}$$

or

$$\begin{aligned}\frac{1}{q_t M_t} &= \alpha + \frac{1}{q_t M_t}\left[\beta\left(1 - \beta\frac{q_{t+1}}{q_t} - \psi_{t+1}^{RE}\frac{\beta}{\alpha}\frac{q_{t+1}}{q_t}\right) + \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}}\right] + \\ &\quad + \psi_t^{RE}\left(\frac{M_t + b_t\frac{\beta^{-1}}{q_t}}{M_t}\right) \qquad (37)\end{aligned}$$

Dividing through by $\alpha,$ and using

$$\frac{1}{\alpha q_t M_t} = 1 + i_t^{RE} = \beta^{-1}\frac{q_t}{q_{t+1}} \qquad (38)$$

we obtain

$$\begin{aligned}1 + i_t^{RE} &= 1 + (1 + i_t^{RE})\left[\beta\left(1 - \frac{1}{1 + i_{t+1}^{RE}} - \frac{\psi_{t+1}^{RE}}{\alpha}\frac{1}{1 + i_{t+1}^{RE}}\right) + \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}}\right] + \\ &\quad + \frac{\psi_t^{RE}}{\alpha}\left(\frac{M_t + b_t\frac{\beta^{-1}}{q_t}}{M_t}\right) \qquad (39)\end{aligned}$$

i.e.,

$$\begin{aligned}1 + i_t^{RE} &= 1 + (1 + i_t^{RE})\left[\beta\left(1 - \left(1 + \frac{\psi_{t+1}^{RE}}{\alpha}\right)\frac{1}{1 + i_{t+1}^{RE}}\right) + \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}}\right] + \\ &\quad + \frac{\psi_t^{RE}}{\alpha}\left(1 + \frac{B_t(1 + i_t^{RE})}{M_t}\right) \qquad (40)\end{aligned}$$

Dividing through by $1 + i_t^{RE}$,

$$1 = \frac{1}{1 + i_t^{RE}} + \beta\left[1 - \left(1 + \frac{\psi_{t+1}^{RE}}{\alpha}\right)\frac{1}{1 + i_{t+1}^{RE}}\right] + \psi_{t+1}^{RE} b_{t+1}\epsilon_{M_{t+1}} + \frac{\psi_t^{RE}}{\alpha}\left[\frac{1}{1 + i_t^{RE}} + \frac{B_t}{M_t}\right] \qquad (41)$$

and so,

$$1=\left(1+\frac{\psi_t^{RE}}{\alpha}\right)\frac{1}{1+i_t^{RE}}+\beta\left[1-\left(1+\frac{\psi_{t+1}^{RE}}{\alpha}\right)\frac{1}{1+i_{t+1}^{RE}}\right]+\psi_{t+1}^{RE}b_{t+1}\epsilon_{M_{t+1}}+\frac{\psi_t^{RE}}{\alpha}\frac{B_t}{M_t} \tag{42}$$

We define, as before,

$$z_t^{RE}\equiv\left(1+\frac{\psi_t^{RE}}{\alpha}\right)\frac{1}{1+i_t^{RE}}$$

and rearrange terms to obtain our difference equation in terms of z_t :

$$z_t^{RE}=(1-\beta)+\beta z_{t+1}^{RE}-\psi_{t+1}^{RE}b_{t+1}\epsilon_{M_{t+1}}-\frac{\psi_t^{RE}}{\alpha}\frac{B_t}{M_t} \tag{43}$$

Can we sign $\epsilon_{b_{t+1}}$? Can we say that $\epsilon_{b_{t+1}}\leq 0$? As long as $b_{t+1}\neq 0$, then $\epsilon_{b_{t+1}}\neq 0$. If $\epsilon_{b_{t+1}}=0$, then when b_{t+1} goes up, q_{t+1} is constant. Since in equilibrium it must be that

$$\frac{\beta}{\alpha}+b_{t+2}-q_{t+1}M_{t+1}-b_{t+1}\beta^{-1}-g=0$$

and

$$M_{t+2}=\frac{\beta}{\alpha}\frac{1}{q_{t+1}}$$

then M_{t+2} is constant and the debt is growing at the rate β^{-1}, violating the budget. This is obvious because all that is being said is that it is not possible to finance a higher debt without ever changing the price levels. However, we cannot make a similar argument for $\epsilon_{b_{t+1}}>0$. Using the present value budget constraint

$$\sum_{s=0}^{\infty}\beta^s\left[\frac{q_{s-1}-q_s}{q_{s-1}}\frac{\beta}{\alpha}-b_t\beta^{-1}-g\right]=0$$

it is clearly possible for the price level to go down, if at some point in the future, the inflation rate goes up in order to balance the budget.

A conjecture for the solution is that the elasticity is negative and the multipliers are decreasing. Also the debt goes to zero and the interest rate converges to the value of multipliers evaluated at the marginal productivity of labor.

We state the main result (conjecture) of the paper:

The optimal time consistent policy satisfies $\psi_t \searrow \underline{\psi}$, $b_t = B_t q_t \searrow 0$ and $(1 + i_t) \searrow (1 + \frac{\underline{\psi}}{\alpha}) = \frac{1}{\beta - \alpha g}$.

Notice that our results amount to saying that, in the case of a recursive equilibrium, z^{RE} converges to 1, which is the solution with indexed debt. Then the nominal interest rate converges to:

$$1 + i^{RE} = \frac{1}{\beta - \alpha g}$$

and the corresponding multiplier converges to:

$$\psi^{RE} = \frac{1}{\beta - \alpha g} - 1$$

In the next section we compare the solution in this recursive equilibrium with the solution obtained when commitment is feasible. Furthermore, we also compute a myopic solution, where the government does not take fully account of the effects of its actions, and compare this to the recursive solution.

5 Other solutions with nominal debt

5.1 The Ramsey solution with full commitment

Let us analyze the Ramsey (R) solutions to the government maximization problem in the case of nominal non zero debt. We solve the problem from date $t = 0$ on.

If the government was able to reoptimize at time $t = 0$, and was able to commit to its policies from there on, then it would want to choose a policy that would differ from the ones obtained before. However, if this had been anticipated it would have to be the case that the ex-ante interest rate equals the ex-post rate. This is the exercise performed by Chari and Kehoe (1999).

Definition 3 *The Ramsey solution is a sequence of quantities and prices such that the government maximizes:*

$$Max \sum_{t=0}^{\infty} \beta^t \left[\ln(q_t M_t) - \alpha(q_t M_t + g)\right]$$

subject to the implementability conditions

$$\frac{\beta}{\alpha} + b_{t+2} - q_{t+1}M_{t+1} - \beta^{-1}b_{t+1} - g = 0,\ t \geq 0 \tag{44a}$$

$$\frac{\beta}{\alpha} + b_1 - q_0 M_0 - q_0 b_0 \frac{\beta^{-1}}{\overline{q}_0} - g = 0 \tag{45}$$

$$M_{t+1} = \frac{\beta}{\alpha}\frac{1}{q_t},\ t \geq 0 \tag{46}$$

Let $\beta^{t+1}\psi_{t+1}^R$, $t \geq 0$, ψ_0^R, and λ_t^R, $t \geq 0$, be the multipliers respectively of the constraints (44a), (45) and (46). The marginal conditions for $q_0, q_{t+1,} M_{t+1}, b_{t+1}$,and the envelope theorem allows us to get:

$$\psi_t^R = \psi^R \tag{47}$$

i.e. constant multipliers as in the case of real debt (even though with different values), and

$$\frac{1}{q_{t+1}M_{t+1}} = \alpha + \frac{\beta}{q_{t+1}M_{t+1}}\left[\frac{1}{q_{t+1}M_{t+2}} - \alpha\frac{q_{t+2}}{q_t} - \psi\frac{q_{t+2}}{q_{t+1}}\right]\frac{\beta}{\alpha} + \psi^R$$

$$\frac{1}{q_0 M_0} = \alpha + \frac{\beta}{q_0 M_0}\left[\frac{1}{q_0 M_1} - \alpha\frac{q_1}{q_0} - \psi\frac{q_1}{q_0}\right]\frac{\beta}{\alpha} + \psi^R\left(\frac{M_0 + b_0\frac{\beta^{-1}}{\overline{q}_0}}{M_0}\right) \tag{48}$$

Rearranging the expressions, we obtain:

$$1 = \frac{1}{1+i_{t+1}^R} + \beta\left[1 - \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1+i_{t+2}^R}\right] + \frac{\psi^R}{\alpha} \tag{49}$$

and again defining

$$z_t^R \equiv \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1+i_t^R}$$

we can rearrange terms to obtain:

$$z_t^R = (1-\beta) + \beta z_{t+1}^R$$

The solution to this difference equation is

$$i^R = \frac{\psi^R}{\alpha} \tag{50}$$

which shows that the interest rate is again equal to the multiplier, calculated at the marginal value of labor, though with a different value for the multiplier, so that the interest rate is also different. This is not valid for some $t = 0$. In fact, the solution allows for a *surprise* inflation at $t = 0$, so that:

$$1 + i_0^R = 1 + \beta(1 + i_0^R)\left[1 - \left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1 + i^R}\right] + \frac{\psi^R}{\alpha}\left[1 + \frac{B_0(1 + \overline{i_0^R})}{M_0}\right] \tag{51}$$

where i_0^R is the ex-post interest rate, corresponding to

$$1 + i_0^R = \frac{u'(co)}{\alpha}$$

while $\overline{i_0^R}$ is the ex-ante interest rate. Naturally, if this was anticipated the two interest rates will have to be equal.

Since

$$\left(1 + \frac{\psi^R}{\alpha}\right)\frac{1}{1 + i^R} = 1$$

(51) becomes:

$$1 + i_0^R = 1 + \frac{\psi^R}{\alpha}\left[1 + \frac{B_0(1 + \overline{i_0})}{M_0}\right] \tag{52}$$

If this solution was anticipated then $\overline{i_0^R} = i_0^R$, in which case we would obtain that the nominal interest rate will be:

$$i^R = \frac{\psi^R}{\alpha} = \frac{i_0^R}{1 + \frac{B_0}{M_0}(1 + i_0^R)} \tag{53}$$

The Ramsey solution is characterized by an interest rate in period 0 that is higher than the one in the solution to the maximization problem with real debt, and a lower rate onwards. The reason is that the government aims at taking advantage of the lump-sum, or apparent lump-sum if anticipated. The government, in fact, has the perception, at some time 0, that it can deplete the nominal liabilities, even if in equilibrium. However, if this is anticipated by the agents, the government cannot effectively deplete the debt.

The feature of the Ramsey problem with nominal debt is that the government can surprise the agents at some $t = t_0$, but what happens if the government was trying to systematically surprise the agents every period? What if the agents anticipated this behavior? If we impose Rational Expectations and analyze this case, we get what in the literature has been called Myopic solution. We are not interested here in the myopic solution itself as an equilibrium concept. However, through this equilibrium concept we can get further insights into the problem. Let us therefore briefly consider this solution.

5.2 A myopic solution without commitment

Let again $q_t \equiv 1/p_t$, and let $\overline{q}_t$ denote the expectations formed at $t-1$ of the price of money at t. This problem is indexed by M.

Definition 4 *A myopic equilibrium is a sequence of quantities and prices such that:*

(i) The government solves the problem:

$$V(M_t, b_t, \overline{q}_t) = \max_{(b_{t+1}, q_t)} \left\{ \ln(q_t M_t) - \alpha(q_t M_t + g) + \beta V(M_{t+1}, b_{t+1}, \overline{q}_{t+1}) \right\}$$

subject to the implementability conditions:

$$\frac{\beta}{\alpha} + b_{t+1} - q_t M_t - q_t b_t \frac{\beta^{-1}}{\overline{q}_t} - g = 0 \tag{54}$$

$$M_{t+1} = \frac{\beta}{\alpha} \frac{1}{q_t} \tag{55}$$

(ii) $q_t = \overline{q}_t$ *all* t.

The marginal condition for q_t is:

$$\frac{1}{q_t} - \alpha M_t + \beta V_{M_{t+1}} \left(-\frac{\beta}{\alpha} \frac{1}{q_t^2}\right) + \beta V_{b_{t+1}} [M_t + \beta^{-1} \frac{b_t}{\overline{q}_t}] = 0 \tag{56}$$

and for b_{t+1}:

$$\beta V_{b_{t+1}} + \psi_t^M = 0 \tag{57}$$

Using the envelope theorem, we get:

$$V_{b_t} = -\psi_t^M q_t \frac{\beta^{-1}}{\overline{q}_t} \tag{58}$$

$$V_{M_t} = \frac{1}{M_t} - \alpha q_t - \psi_t^M q_t \tag{59}$$

where variables are defined as above. Using (57) and (58) and imposing the consistency condition $q_t = \overline{q}_t$, all t, gives:

$$\psi_t^M = \psi_{t+1}^M = \psi^M \tag{60}$$

Dividing through (54) by M_t and rearranging, using (55), we get:

$$\frac{1}{q_t M_t} = \alpha + \frac{\beta}{q_t M_t}\left[\frac{\alpha}{\beta} - \alpha\frac{q_{t+1}}{q_t} - \psi_{t+1}^M \frac{q_{t+1}}{q_t}\right]\frac{\beta}{\alpha} + \psi_t^M \left(\frac{M_t + b_t \frac{\beta^{-1}}{q_t}}{M_t}\right) \tag{61}$$

Dividing through by α, and using

$$\frac{1}{\alpha q_t M_t} = 1 + i_t^M = \beta^{-1}\frac{q_t}{q_{t+1}} \tag{62}$$

we obtain

$$1 + i_t^M = 1 + \beta(1 + i_t^M)\left[1 - \left(1 + \frac{\psi_{t+1}^M}{\alpha}\right)\frac{1}{1 + i_{t+1}^M}\right] + \frac{\psi_t^M}{\alpha}\left[1 + \frac{B_t(1 + i_t^M)}{M_t}\right] \tag{63}$$

Dividing through by $1 + i_t^M$,

$$1 = \frac{1}{1 + i_t^M} + \beta\left[1 - \left(1 + \frac{\psi_{t+1}^M}{\alpha}\right)\frac{1}{1 + i_{t+1}^M}\right] + \frac{\psi_t^M}{\alpha}\left[\frac{1}{1 + i_t^M} + \frac{B_t}{M_t}\right]$$

Let us again define:

$$z_t^M \equiv \left(1 + \frac{\psi_t^M}{\alpha}\right)\frac{1}{1 + i_t^M}$$

and rearrange terms to obtain

$$z_t^M = (1 - \beta) + \beta z_{t+1}^M - \frac{\psi_t^M}{\alpha}\frac{B_t}{M_t}$$

From (60) we know that $\psi_t^M = \psi^M$. This can be written as

$$z_{t+1}^M = \frac{z_t^M}{\beta} + \frac{1}{\beta}\left[\frac{\psi^M}{\alpha}\frac{B_t}{M_t} - (1-\beta)\right]$$

If $\frac{\psi^M}{\alpha}\frac{B_t}{M_t} - (1-\beta) > 0$, then z^M would be explosive, and the interest rate would become negative. If $\frac{\psi^M}{\alpha}\frac{B_t}{M_t} - (1-\beta) < 0$, then the only solution is $\frac{B_t}{M_t}$ constant and

$$z_t^M = z^M = 1 - \frac{1}{(1-\beta)}\frac{\psi^M}{\alpha}\frac{B_t}{M_t} \tag{64}$$

Hence, since we assume throughout that $B_t > 0$, $z^M < 1$ and constant.

Note that, when the nominal debt is zero, $B_t = 0$, (64) implies:

$$i^M = \frac{\psi^M}{\alpha}$$

This means that, in this case, the multiplier of the implementability condition is higher without nominal debt and coincides with the one with indexed debt.

Since the solution is stationary, we define:

$$b = B_0 q_{-1} = B_0 q^*$$

and write the expression for the multipliers as

$$\frac{\psi^M}{\alpha} = \frac{i^M}{1 + \frac{1}{(1-\beta)}\frac{B_t}{M_t}(1+i^M)} \tag{65}$$

and the solution for the nominal interest rate (given by the budget constraint (54), as:

$$1 + i^M = \frac{1}{\beta - \alpha g - \alpha b\frac{1-\beta}{\beta}} \tag{66}$$

This can be interpreted as follows: Since the solution is stationary, the nominal interest rate i is the rate that balances the budget for a given stationary real debt. The marginal value of the distortion is discounted by the present value of the existing debt (the term $\frac{1}{(1-\beta)}\frac{B_t}{M_t}(1+i^M)$), that is a function of the nominal debt. This accounts for the perception of lump sum taxation on the nominal debt, i.e. that it is possible, ex-post, decrease the real value of debt by inflating. Hence the multipliers are lower.

In our environment it is possible to vary the inflation tax over time but there are no alternative taxes. If there were alternative taxes, in this environment they would be equivalent to the inflation tax. However if there were cash and credit goods, and the utility function was the same on the two goods, maintaining the separability in leisure, then although the optimal inflation tax would be the Friedman rule, that would not be the solution in this myopic equilibrium. There would be a deviation from the Friedman rule, since the inflation tax would have this apparently lump sum component associated with the time inconsistency, that would induce an inflation tax that would be too high.

As we said, the above policy is myopic since the effects of policy on the future policy variables are not taken into account. In the recursive equilibrium we have discussed above these effects had been incorporated.

6 A Comparison of the results.

Let us now compute the recursive solutions[7], summarize our analytical and quantitative results, and make some policy comparisons. Recall that we have used the following notation: superscript I = Myopic policy with Indexed Debt, M = Myopic policy with Nominal Debt, R = Ramsey, RE = markov perfect or REcursive policy.

Let us start by comparing the zs:

In the solution with indexed debt (I):

$$z^I = 1$$

In the myopic solution with nominal debt, the value of z is still constant but:

$$z_t^M = (1-\beta) + \beta z_{t+1}^M - \frac{\psi_t^M}{\alpha} \frac{B_t}{M_t} < 1$$

As far as the Ramsey case is concerned, we know that the value of z is lower that the indexed case from $t = 0$ to $t = 1$, but coincides with the indexed case afterwards (cf. Graph 1). In the recursive solution,

$$z_t^{RE} = (1-\beta) + \beta z_{t+1}^{RE} - \psi_{t+1}^{RE} b_{t+1} \epsilon_{M_{t+1}} - \frac{\psi_t^{RE}}{\alpha} \frac{B_t}{M_t} \leq 1$$

[7] See the Appendix for the specification of the algorithm to compute the optimal policy in the recursive equilibrium.

and z_t^{RE} converges to one. Note that this equation differs from the corresponding equation in the myopic equilibrium by the term $-\psi_{t+1}^{RE} b_{t+1} \epsilon_{M_{t+1}}$, as well as the equation for the multiplier $\psi_t^{RE} = \left(1 - \epsilon_{b_{t+1}}\right) \psi_{t+1}^{RE}$, where we have the elasticity of the policy function $Q(M, b)$ with respect to the debt.

We briefly mention above that from the equilibrium budget constraint, we can calculate the values of the interest rates (and the multipliers). More specifically, we know that in the case of indexed debt and in the case of the myopic policy with rational expectations imposed from $t = -1$, we have that:

$$1 + i^I = 1 + i^M = \frac{1}{\beta - \alpha g - \alpha b \frac{1-\beta}{\beta}}$$

Since both problems are stationary, the nominal interest rate is the same in both cases and is the one that allows to finance the stationary debt and government expenditures.

When the debt is zero we have:

$$1 + i = \frac{1}{\beta - \alpha g}$$

which is still constant but higher (other things equal).

We also know from the above analysis that:

$$i^R < i^M = i^I$$

from period $t = 1$ onwards, but higher at $t = 0$. Hence, as far as the Ramsey interest rate is concerned, we know that it starts at a level higher than the indexed and myopic and that then it decreases at $t = 1$, being constant and lower than the indexed and myopic cases.

In the case of a recursive policy the equilibrium interest rate is non constant. At the beginning:

$$i_0^{RE} > i^M = i^I$$

where in the denominator we have the present value of the existing debt, which accounts for the perception of lump sum taxation in the nominal debt. However, also in this case, the interest rate decreases and converges to:

$$1 + i^{RE} = \frac{1}{\beta - \alpha g} \tag{67}$$

Hence, we know that the interest rate when the REcursive policy is followed starts at a higher value. But, in the limit, since the debt is depleted it goes down so that,

$$i^{RE} < i^M = i^I$$

i.e. it converges to the level of the interest rate i when debt is zero.

Finally, when we compare the multipliers ψ, it is easy to see that:

$$\frac{\psi^I}{\alpha} = i^I$$

$$\frac{\psi^M}{\alpha} = \frac{i^M}{1 + \frac{B_t}{M_t}\frac{1+i^M}{1-\beta}}$$

$$\begin{aligned} \frac{\psi^R}{\alpha} &= \frac{i_0^R}{1 + \frac{B_0}{M_0}(1 + i_0^R)} \\ \frac{\psi^R}{\alpha} &= i^R \end{aligned}$$

Hence:

$$\psi^R < \psi^I$$

The multiplier ψ^R is lower when debt is non indexed, because the stationary interest rate is lower and because, although the initial interest rate (i_0^R) is higher, the multiplier is discounted by the *apparent lump sum effect.* Since (from budget)

$$i^I = i^M$$

we have that:

$$\psi^I > \psi^M$$

$$\psi^{RE} > \psi^I$$

In the markov perfect policy, however, the multiplier converges to:

$$\psi^{RE} = \frac{1}{\beta - \alpha g} - 1 \tag{68}$$

In the transition we have the following expression:

$$
\begin{aligned}
\frac{\psi_{t+1}^{RE}}{\alpha} &= \\
&= \frac{i_t^{RE} + \frac{\beta}{1-\beta}\left[\frac{1+i_t^{RE}}{1+i_{t+1}^{RE}} - 1\right]}{\frac{1}{1-\beta}\left[1 - \beta\frac{1+i_t^{RE}}{1+i_{t+1}^{RE}} + \frac{B_t}{M_t}(1+i_t^{RE})\right] + (1+i_t^{RE})\beta\frac{B_{t+1}}{M_{t+1}}\epsilon_{M_{t+1}} + -\epsilon_{b_{t+1}}\left(1 + \frac{B_t}{M_t}(1+i_t^{RE})\right)}
\end{aligned}
$$

We are not able to compare exactly ψ^{RE} with ψ^M.

Notice that the solution of a constant interest rate obtained in the case of indexed debt (I) is time consistent, while the solution of a constant interest rate in the myopic equilibrium with nominal debt (M) is not time consistent. If we imposed that the Ramsey had to be stationary, then we would be back to the same solution as before, with different multipliers though. These differing multipliers suggest the solution that would be obtained if the environment was modified to allow for a trade-off between the inflation tax and an alternative tax. We can say that if the government debt is indexed, the same optimal monetary policy is implemented with and without full commitment. However, with nominal debt, the full commitment policy is not time consistent, although interest rates are stationary after period zero. Because the government is tempted to inflate to deplete the debt, a time-consistency problem arises. However, eventually it vanishes.

Let us now compare the two solutions with indexed and nominal debt for the same real debt. Since both problems are stationary, the nominal interest rate is the same in both cases and is the one that allows to finance the stationary debt and government expenditures. The difference in the two solutions is that the multiplier is lower when the debt is nominal. If there were alternative taxes, this would induce a too high inflation tax. In our very simple framework there are no welfare effects of the alternative regimes. Welfare comparisons, because of the simple environment we have analyzed, are not meaningful.

7 Conclusions

This paper addresses the issue of how debt, and in particular nominal debt, affects the choice of the optimal monetary policy and shows that

- If there is no inherited debt, the interest rate is constant and the optimal monetary policy is time consistent.
- If the debt is indexed, the same optimal monetary policy is implemented with and without full commitment.
- With nominal debt, instead, the full commitment policy is not sequential. A time consistency problem arises: the government is tempted to inflate away its nominal liabilities.
- With nominal debt, without full commitment, the optimal sequential policy has the feature that debt is progressively depleted.

8 Appendix 1: Computing the recursive equilibrium I

This is the description of the algorithm to solve for the optimal monetary policy.

Let $q_t = Q(M, b)$, the first order condition of the recursive problem 29 becomes:

$$\frac{1}{q_n}\left[1 - \beta W_{M_{t+1}} M_{t+1} + W_{b_{t+1}} b\right] - \left(\alpha - \beta W_{b_{t+1}}\right) M_t = 0$$

from which we can get the expression to compute $Q(M, b)$:

$$Q(M,b) = \frac{[(1 - \beta W_M(M', b')M' + W_b(M', b')b)]}{(\alpha - \beta W_b(M', b'))\, M}$$

which in the algorithm takes the form:

$$Q^{n+1}(M,b) = \frac{[(1 - \beta W_M^n(M', b')M' + W_b^n(M', b')b)]}{(\alpha - \beta W_b^n(M', b'))\, M}$$

where:

$$W_M^n(M', b') = \frac{1}{M} - \alpha Q(M,b) - \psi Q(M,b) + V_Q \frac{\partial Q(M,b)}{\partial M}$$

which after substitutions becomes:

$$\begin{aligned} W_M^{n+1}(M,b) &= \frac{1}{M} - \alpha Q^{n+1}(M,b) - \\ & \quad W_b^n(M', b') \left[\frac{b}{\frac{b}{M}\frac{M}{Q(M,b)}} \frac{\partial Q^{n+1}(M,b)}{\partial M} - \beta Q^{n+1}(M,b) \right] \end{aligned}$$

$$W_b^{n+1}(M', b') = \left(1 - \epsilon_b^{Q^{n+1}}(M,b)\right) W_b^n(M', b')$$

Initial conditions:

$$Q^0(M,b) = \frac{1}{M}\left(\bar{c} - b\beta^{-1}\right) \quad (69)$$
$$W_b^0(M,b) = -\beta^{-1}c_n + g \quad (70)$$
$$b_{n+1} = c_n + b_n\beta^{-1} + g - \frac{\beta}{\alpha} \quad (71)$$
$$M_{n+1} = \frac{\beta}{\alpha}\frac{1}{q_n} \quad (72)$$

However, since the function V is not known, we follow an iterative process to obtain it.

1. **Step 1 (T+1):**Corresponds to the steady state solution without any remaining debt:

Equilibrium conditions for Step 1:

$$\begin{aligned} c_{T+1} &= c_\infty = \frac{\beta}{\alpha} - g \\ n_{T+1} &= \frac{\beta}{\alpha} \\ b_{T+2} &= b_\infty = 0 \\ M_{T+2} &= \frac{\beta}{\beta - \alpha g} M_{T+1} \end{aligned}$$

1. **Step 2 (T):**Since $b_{T+1} = 0$,

$Q_T(M_T, b_T) = \frac{(\frac{\beta}{\alpha} - g) - b_T\beta^{-1}}{M_T}$ and we can obtain $(c_T, n_T, b_{T+1}, M_{T+1})$ using (**??**)- (72). Furthermore, we can also obtain the corresponding value function:

$$W_T(M_T, b_T) =$$

$$= \log((\frac{\beta}{\alpha} - g) - b_T\beta^{-1}) - \alpha(\frac{\beta}{\alpha} - b_T\beta^{-1}) + \beta W_\infty$$

as well as its derivatives:

$$\begin{aligned} \frac{\partial W_T}{\partial M_{n+1}} &= 0 \\ \frac{\partial W_T}{\partial b_{n+1}} &= -(\frac{1}{c_T} - \alpha)\beta^{-1} \end{aligned}$$

It follows that, at this stage, is easy to find $q_{T-1} = Q_{T-1}(M_{T-1}, b_{T-1})$ analytically solving (**??**),

$$[\frac{1}{q_{T-1}} - \alpha M_{T-1}] - [\frac{1}{c_T} - \alpha][M_{T-1} + \beta^{-1}\frac{b_{T-1}}{q_{T-1}}] = 0$$

that is,

$$\begin{aligned} q_{T-1} &= \left[c_T - (1-\alpha c_T)\,\beta^{-1} b_{T-1}\right]\frac{1}{M_{T-1}} \\ &= \end{aligned}$$

and, using

$$q_{T-1} = \frac{1 - b_{T-1}\beta^{-1}}{M_{T-1}}[(\frac{\beta}{\alpha} - g) - \beta^{-1}[M_{T-1}q_{T-1} + b_{T-1}\beta^{-1}] - (\frac{\beta}{\alpha} - g)]$$

Equilibrium conditions for Step 2:

$$\begin{aligned} c_T &= \frac{\beta}{\alpha} - g - b_T\beta^{-1} \\ n_T &= \frac{\beta}{\alpha} - b_T\beta^{-1} \\ b_{T+1} &= 0 \\ M_{T+1} &= \frac{\beta}{\alpha}\frac{1}{q_T} \\ M_T q_T &= c_T \end{aligned}$$

1. **Step 3 (T-1):**

$$W_{T-1}(M_{T-1}, b_{T-1}) = V_{T-1}(M_{T-1}, b_{T-1}, Q_{T-1}(M_{T-1}, b_{T-1})) =$$

$$\max_{q_{T-1}} \left\{ \begin{array}{c} \log[M_{T-1}Q_{T-1}(M_{T-1}, b_{T-1}) - \alpha(M_{T-1}Q_{T-1}(M_{T-1}, b_{T-1}) + g) + \\ \beta\{\log[(\frac{\beta}{\alpha} - g) - M_{T-1}Q_{T-1}(M_{T-1}, b_{T-1})]\beta^{-1} - \alpha(\frac{\beta}{\alpha} - M_{T-1}Q_{T-1}(M_{T-1}, b_{T-1})\beta^{-1} \end{array} \right.$$

$$\begin{aligned} W_{M_T} &= V_{T1} + V_{T3}\frac{\partial Q}{\partial M_{T-1}} = [(\frac{1}{c_{T-1}} - \alpha) - (\frac{1}{c_T} - \alpha)]q_{T-1}(1 + \epsilon_{Q_{T-1}M_{T-1}}) \\ W_{b_T} &= V_{T2} + V_{T3}\frac{\partial Q}{\partial b_{T-1}} = [(\frac{1}{c_{T-1}} - \alpha) - (\frac{1}{c_T} - \alpha)]\frac{c_{T-1}}{b_{T-1}}\epsilon_{Q_{T-1}b_{T-1}} \end{aligned}$$

Equilibrium conditions for Step 3 (for $c_{T-1}, n_{T-1}, b_T, M_T; q_{T-1}, \overline{q}_{T-1}$) :

$$\left(\frac{1}{c_{T-1}} - \alpha\right) M_{T-1} - \left(\frac{1}{c_T} - \alpha\right) b_T \left(M_{T-1} + b_{T-1}\beta^{-1}\frac{1}{\overline{q}_{T-1}}\right) = 0$$

$$\begin{aligned} c_{T-1} &= q_{T-1}M_{T-1} \\ n_{T-1} &= c_{T-1} + g \\ b_T &= c_{T-1} + b_{T-1}\beta^{-1} + g - \frac{\beta}{\alpha} \\ M_T &= \frac{\beta}{\alpha}\frac{1}{q_{T-1}} \\ \overline{q}_{T-1} &= q_{T-1} \end{aligned}$$

1. **Step 4 (T-n):**

$$W_{T-n}(M_{T-n}, b_{T-n}) = V_{T-n}(M_{T-n}, b_{T-n}, Q_{T-n}(M_{T-n}, b_{T-n})) =$$

$$\begin{aligned} &\max_{q_{T-n}}\{\log(q_{T-n}M_{T-n}) - \alpha(q_{T-n}M_{T-n} + g) + \\ &+\beta W_{T-n+1}(\frac{\beta}{\alpha}\frac{1}{q_{T-n}}, q_{T-n}M_{T-n} + b_{T-n}\beta^{-1}\frac{q_{T-n}}{\overline{q}_{T-n}} + g - \frac{\beta}{\alpha})\} \end{aligned}$$

$$\overline{q}_{T-n} = q_{T-n} = Q_{T-n}(M_{T-n}, b_{T-n})$$

Equilibrium conditions for Step 4:

$$\left(\frac{1}{c_{T-n}} - \alpha\right) M_{T-n} - \left(\frac{1}{c_{T-n+1}} - \alpha\right) b_{T-n+1} \left(M_{T-n} + b_{T-n}\beta^{-1}\frac{1}{\overline{q}_{T-n}}\right) = 0$$

$$\begin{aligned} c_{T-n} &= q_{T-n}M_{T-n} \\ n_{T-n} &= c_{T-n} + g \\ b_{T-n+1} &= c_{T-n} + b_{T-n}\beta^{-1} + g - \frac{\beta}{\alpha} \\ M_{T-n+1} &= \frac{\beta}{\alpha}\frac{1}{q_{T-n}} \\ \overline{q}_{T-n} &= q_{T-n} = Q_{T-n}(M_{T-n}, b_{T-n}) \end{aligned}$$

References

[1] Alesina, Alberto and Roberto Perotti. 1996. "Fiscal Adjustments in OECD countries: composition and macroeconomic effects", IMF WP 96/70, July.

[2] Beetsma, Roel M.W. and Uhlig, Harald. 1997. "An Analysis of the 'Stability Pact'," CEPR Disc. Paper No. 1669.

[3] Buiter, Willem, Giancarlo Corsetti and Nouriel Roubini. 1993. "Excessive deficits: Sense and Nonsense in the Treaty of Maastricht," *Economic Policy*, 8, 57-100.

[4] Canzoneri, Matthew B. and Behzad T. Diba. 1991."Fiscal deficits, financial integration, and a central bank for Europe, *Journal of Japanese and International Economics*, 5, 381-403.

[5] Chari, V.V. and Patrick J. Kehoe. 1997. "On the Need for Fiscal Constraints in a Monetary Union," (mimeo) Federal Reserve Bank of Minneapolis.

[6] Chari, V.V. and Patrick J. Kehoe. 1999. "Optimal Fiscal and Monetary Policy", National Bureau of Economic Research, WP 6891, January.

[7] Cochrane John, 1998, "Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level", National Bureau of Economic Research, WP 6771, January.

[8] Dornbusch, Rudi. 1997. "Fiscal Aspects of Monetary Integration", *American Economic Review P&P*, 87, 221-223.

[9] Evans, George W., Seppo Honkapohja and Ramon Marimon. 1997. "Fiscal Constraints and Monetary Stability" (mimeo) European University Institute.

[10] Gertler, M. and S. Gilchrist. 1994. "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms," *Quarterly Journal of Economics,* CIX(2), 309-340.

[11] Giovannetti, Giorgia and Ramon Marimon. 1999. "An EMU with Different Transmission Mechanisms?"in J. Von Hagen and C. waller eds, **Common Money, Uncommon regions**, Kluwer. .

[12] Giovannetti, Giorgia, Ramon Marimon and Pedro Teles, 1998, "Policy Commitments and the Creation of EMU" in J. Arrowsmith ed, **Thinking the Unthinkable about EMU**, National institute of Economic and Social Research, London.

[13] Lucas, Robert E., Jr. and Nancy L. Stokey, 1983, "Optimal Fiscal and Monetary Theory in an Economy without Capital", *Journal of Monetary Economics,* 12, 55-93.

[14] Marimon, Ramon. 1997. "The Fiscal Theory of Money as an Unorthodox Financial Theory of the Firm." (mimeo) European University Institute.

[15] Marimon, Ramon, Juan Pablo Nicolini and Pedro Teles, 1997. "Electronic Money: the end of inflation?" Institute for Empirical Macroeconomics Discussion Paper 122, Federal Reserve Bank of Minneapolis.

[16] Nicolini, Juan Pablo, 1997, "More on the Time Inconsistency of Optimal Monetary Policy", *Journal of Monetary Economics*, forthcoming.

[17] Sargent, Thomas, J. and Neil Wallace, 1981, "Some Unpleasant Monetarist Arithmetic," *Quarterly Review,* Federal Reserve bank of Minneapolis, 5,3, 1-17. [Reprinted, 1993, in *Rational Expectations and Inflation, 2nd ed.,* New York, Harper Collins].

[18] Sims, Christopher A. , 1994, "A Simple Model for the Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy", *Economic Theory*, 4, 381-399.

[19] Svensson, L.E.O., 1985, "Money and Asset Prices in a Cash-in-Advance Economy", *Journal of Political Economy*, 93, 919–944.

[20] Woodford, Michael. 1996. "Control of the Public Debt: A Requirement for Price Stability? NBER, Working Paper 5684

[21] Woodford, Michael. 1996. "Loan Commitments and Optimal Monetary Policy? NBER, Working Paper 5660.